

TEKNILLINEN KORKEAKOULU  
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## Calculating margin requirements for derivative clearing accounts

Diplomi-insinöörin tutkintoa varten tarkastettavaksi jätetty diplomityö.

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<p>Johdannaiskaupassa kaupan osapuolille syntyy erilaisia oikeuksia ja velvollisuuksia. Kauppapaikkana ja molempien osapuolien vastapuolena selvitysyhtiön ominaisuudessa toimiva pörssi kattaa omat riskinsä vaatimalla velvollisuuksien kohteena olevilta osapuolilta vakuuksia. Vakuusvaatimuksen laskemisessa pyritään kattamaan tiliaseman todennäköinen riski ja toisaalta huolehtimaan, ettei asiakkaalta vaadita liikaa vakuuksia.</p> <p>Vakuusvaatimuksen laskemiseen käytetään tilastollisia malleja, joilla pyritään ennustamaan tiliaseman arvon mahdollisia muutoksia. Vakuusvaatimus on tiliaseman suurin mahdollinen sulkemiskustannus tietyllä riskitasolla.</p> <p>Tässä työssä esitellään Helsingin pörssissä käytetyt vakuuslaskentamenetelmät, vakuuslaskennassa käytetyt algoritmit sekä vakuuslaskentaohjelmiston toteutus.</p>		
Avainsanat: Johdannaiskauppa, johdannaisselvitys, vakuuslaskenta.		



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ABSTRACT OF  
MASTER'S THESIS

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<p>A derivative trade creates different rights and obligations for the trading parties. The exchange, acting as the market place and as the counterparty for both parties as a clearing house, covers its own risks by requiring margin payments from parties with obligations. Margin calculation aims to cover the likely risk of the position and on the other hand ensure that the customer is not overburdened with unnecessary margin requirements.</p> <p>Margin calculation uses statistical models, which try to predict possible changes in the value of the position. The margin requirement is the greatest possible cost to close the position at a specified risk level.</p> <p>In this thesis the structure and margining mechanisms used at Helsinki Exchanges are documented, along with the algorithms used in margin calculation and their software implementation.</p>		
<b>Keywords:</b> Derivative trading, derivative clearing, margin calculation.		



## Foreword

This thesis was written at Helsinki Exchanges in conjunction with the HEXBos project, whose aim was to implement clearing and back-office functionality for Finnish derivatives traded in Eurex.

I would like to thank my supervisor, Prof. Olli Simula, and my instructor, MSc (Tech.) Tomi Palkama, for their help in getting this thesis written, and HEX Ltd. for the possibility to write this thesis.

I would also like to thank the Trappist monks of the Abbey of Scourmont for making the world a better place to live in.

Otaniemi, 18 May 2000



Olli-Pekka Rinta-Koski



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## Symbols and abbreviations

$C$	Price of call option
$K$	Strike price; option exercise price
$N(z)$	Cumulative standard normal distribution function
$P$	Price of put option
$r$	Interest rate
$S$	Spot price
$T$	Maturity; time of expiration of derivative
$\Delta$	Delta; partial derivative of option price in relation to underlying price
$\eta$	Eta; partial derivative of option price in relation to option volatility
$\mu$	Expected rate of stock return
$\Pi$	Value of combined position (portfolio)
$\phi(m, s)$	Normal distribution with mean $m$ and standard deviation $s$
$\sigma$	Volatility; variance of stock return
HEX	Helsinki Exchanges
OCI	Oracle Call Interface, a database API
ODBC	Open Database Connectivity, a database API
OTC	Over-the-counter
XML	Extensible Markup Language



# Chapter 1

## Introduction

A common misconception about risk management in some quarters is that the use of derivative securities constitutes speculation—that is, the addition of financial risk to the business risk of a firm's operations. Some folks think that condom use increases risk. — *Walter Dolde*[5]

### 1.1 Background

Derivative contract parties have various responsibilities. The exchange acts as a central counterparty in all derivative deals to all parties. Derivative contracts are thus considered risk-free<sup>1</sup>, as the exchange is legally bound to maintain sufficient collateral to cover these responsibilities. This collateral is needed to close the position in case a contract party does not fulfill their obligations.

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<sup>1</sup>Only regarding counterparty risk, of course.



To cover its own risk position, the exchange calculates margin requirements for each derivative clearing account and requires collateral deposits from the owners of those accounts where the overall margin requirement for all the contracts in the account is negative.

## 1.2 Stating the problem

Helsinki Exchanges, hereafter called HEX, has been using proprietary software for derivative clearing, maintaining the derivative accounts and calculating margin requirements. This software was developed in-house and has been in use since 1988 when derivative trading in Finland began. In 1998 HEX and Eurex, the derivative exchange in Frankfurt, announced a trading alliance. The alliance is intended to enable all HEX members to trade in Eurex products and vice versa. From a software point of view this means that all HEX products will be transferred to the trading system of Eurex.

As a result of the software change, some of the product descriptions will change as the Eurex trading system is not able to accommodate all products as is. One of the major changes is the conversion of forwards to futures as there are no forward products on Eurex. Also, some of the products traded on Eurex have not been traded on HEX, for example options on futures. Since Eurex does not provide individual margin calculations for customer accounts, HEX has decided to continue offering this service for member brokerages. These changes, along with other changes resulting from the transfer, necessitated a complete overhaul



and partial re-design of the margin calculation system used.

## 1.3 Goals

The goals of this thesis are twofold. The first goal is to formalize and document derivative margin calculation and the procedures used at HEX. The second goal is to adapt existing procedures and develop new ones for margin calculation of Eurex products. The software implementation is described in detail in chapter 4.

## 1.4 Scope

The scope of this thesis is margin calculation for those products where the so-called volatility matrix based margining is used as described in section 3.1. Other methods for calculating margin requirements, such as yield curve based margining and binomial models, are described briefly in later sections of chapter 3.



# Chapter 2

## Definitions

The errors of definitions multiply themselves according as the reckoning proceeds; and lead men into absurdities, which at last they see but cannot avoid, without reckoning anew from the beginning.

— *Thomas Hobbes*[15]

This chapter defines some terms that are essential in understanding later chapters.

### 2.1 Market participants

#### Exchange

An *exchange* is a market place where bid and ask offers from trading parties are matched into trades.



## Trading party

A *trading party* is a brokerage that is trading on the exchange either on its own behalf or on behalf of its customers. Individual investors cannot trade directly on the exchange.

## Clearing house

A *clearing house* is the company handling payments and other settlements resulting from trade contracts made in the exchange. In the case of HEX the exchange itself is also a clearing house in both stock and derivative markets.

## Clearing party

A *clearing party* is a company that handles the clearing transactions on behalf of trading parties. Some trading parties are also clearing parties, but this is not true in general.

## Custodian

A *custodian* is an organization approved by the clearing house to manage assets delivered by clearing parties as collateral deposits in order to fulfill margin requirements. HEX offers a custodian service, but there are also some larger banks that have been granted the right to work as custodian on behalf of HEX.



## 2.2 Instruments

### Derivative

A *derivative* is a financial instrument whose value depends on the values of other, more basic underlying variables. Traditionally derivatives were contracts traded by different financial institutions with the specifics of each contract agreed to separately each time a new contract was made. These contracts are called *over-the-counter* or *OTC* derivatives.

OTC derivative contracts have two basic problems, both of which are addressed by derivatives available from derivative markets. The first is the fact that the per-contract flexibility is also a burden when a contract holder wants to close their position (see 2.4) since there might not be any closing contract available. The second is the variable risk of *default*—the failure of a contract party to fulfill their obligations—because each contract made with a different counterparty poses a different degree of risk.

The problem of closing a position is solved by using standardised derivative contracts. The derivative exchange offers a set of standardised derivative series for trading. This set is composed with the intent of providing enough liquidity so that both opening and closing transactions can be made.

The second problem is solved by the clearing house acting as a central counterparty. All trades effectively result in two contracts: one between the buyer and the clearing house, one between the seller and the clearing house. The clearing



house is legally bound to maintain sufficient collateral to cover possible defaults.

Unlike stocks or other such securities, there is no set amount of derivative contracts on the market. Instead, every time a buyer and a seller agree on the price, a new contract is created. The number of outstanding contracts in a derivative is called *open interest*.

## Option

*Options* are derivatives which give the buyer the right—not an obligation—to buy or sell the underlying asset at the option exercise price (see 2.3). The seller (also called the writer) of an option has an obligation to sell (or buy). Options which give the buyer the right to buy the underlying are called *call options* and options which give the right to sell the underlying are called *put options*. The buyer pays a *premium* to the writer for the right given by the option contract at the time the contract is made.

Options that have intrinsic value—call options that entitle the holder to buy the stock at a lower than market price, for instance—are called *in-the-money* options. Options that have no intrinsic value and thus will expire as worthless, unless the market changes favourably—call options with a higher-than-market exercise price—are called *out-of-the-money* options. The option series that has the closest exercise price to the current market price is called *at-the-money*. Call options with very small exercise prices and put options with very large exercise prices are called *deep-in-the-money* options, and likewise call options with very



large exercise prices and put options with very small exercise prices are called *deep-out-of-the-money* options.

The difference between intrinsic value and option price is called *time value*. This difference is due to the volatility of the underlying—if stocks A and B both have a spot price of 100, call options on both at strike price 90 have an intrinsic value of 10, but if A is highly volatile (the price of A is fluctuating wildly) while B is not, the option on A has a higher time value as A is more likely to appreciate than B<sup>1</sup>, and so the option on A will be more expensive.

Option contracts are made available by the exchange by opening *option series* for trading. Each series has four major distinguishing features: call or put, underlying, strike price and expiration day. These can typically be read from the trading code of the option. Example: the option series with the trading code SRA1V0F60 is a call option, has SRA1V (the trading code for Sonera stock) as the underlying, has a strike price of 60 euros and expires in June 2000.

## Future

In the general sense, a *future* is a contract covering the sale of financial instruments or physical commodities for future delivery. However, these instruments may be notional, as is the case with index futures (see 2.2).

Futures are *marked-to-market* daily. The daily settlement procedure calculates

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<sup>1</sup>Of course, A is also more likely to depreciate, but in that case the option will expire as worthless and no additional harm is done.



net losses and profits to the receivable and payable accounts of the counterparties.

## Forward

*Forwards* are similar to futures, with the difference that there is no daily settlement. Forwards are settled only on settlement day. In the case of stock forwards, final settlement means the physical delivery of the underlying stock. All payments involving forward contracts occur on settlement day.

## Underlying

All derivative contracts have an underlying asset, of which there are several types. Derivative market prices are typically strongly linked to the price<sup>2</sup> of the underlying—if the underlying appreciates by one unit, an in-the-money option on the underlying will appreciate by the same amount, if the effect of volatility is not taken into account. All rights and obligations given by derivatives are directed towards the underlying.

## Stock

Stock derivatives have a stock as their underlying. The buyer of a stock call option, should he decide to exercise it, is entitled to buy as many of the underlying stock as the contract size of the option indicates. Exercise of a stock option usually

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<sup>2</sup>Or equivalent, like an index value.



results in *physical delivery* of the actual stocks, i.e. ownership of the underlying stock changes.

### **Index**

Index derivatives are not based on individual stocks. Their underlying is a notional asset, valued with the aid of an index. The index composition is called the *index basket*. Since the underlying asset is not real, index derivatives do not result in physical delivery. Instead the balance of an in-the-money option is paid to the account of the option holder.

### **Currency**

Currency derivatives are based on currency rates and are always notional—there is no physical delivery of the underlying currency, the derivative is settled as with index derivatives.

### **Interest rate**

Interest rate derivatives are another type of derivative with a notional underlying, in this case the interest rate used in lending by a bank or another financial institution. Strictly speaking, the underlying of an interest rate derivative is a specific loan, either real or imaginary.



## Future

While futures are derivative contracts, they can also be underlyings for other derivatives, usually options. Options on futures are used mainly when the underlying of the future is either not traded or has low liquidity.

## 2.3 Characteristics

### Contract size

*Contract size* is a parameter of the option contract and indicates how many underlying instruments the contract refers to. For instance, the buyer of a single STOX<sup>3</sup> stock call option is entitled to buy 10 underlying shares.

### Strike price

*Strike price*, also referred to as *exercise price*, is the price at which the buyer of an option is entitled to buy or sell the underlying, i.e. exercise the option. A STOX call option with a strike price of 100 euros entitles the option holder to buy 10 underlying shares for 100 euros each.

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<sup>3</sup>STOX is a trademark of HEX and is the marketing name of HEX stock option products.



## Expiration day

*Expiration day* is the last day when the derivative contract gives rights and obligations, i.e. when the contract reaches *maturity*. For options, it is the last day when it is possible for the holder of an option to exercise the rights given by the option.

There are two major types of options which differ only in regard to the significance of expiration day. *European-style options* can only be exercised on expiration day. *American-style options* can be exercised on any day between the first settlement day of the series and expiration day, inclusive. These terms can be a bit misleading as most options on both sides of the Atlantic are American-style.

## Volatility

*Volatility* is a measure of risk based on the standard deviation of the underlying asset price. It indicates the likelihood of price fluctuation. There are three ways to calculate the volatility of an asset.

**Historical volatility** is calculated from underlying market prices for a number of days in the past. It can be calculated for any period of time, but since it uses actual market prices, it can only go backwards in time.

**Implicit volatility** is calculated from option market prices. A pricing model is used to translate the price of an option into a prediction of future volatility.



Implicit volatility is a measure of market expectations of future changes in the value of the underlying asset. A large implicit volatility means that there is a big uncertainty of the future value of the underlying on the market.

**Market volatility** is calculated as the average of implicit volatilities for all different option series with the same underlying.

## Delta

Delta is a measure of change in the value of an option when the value of the underlying changes. In other words, the delta of a call option,  $\Delta_c$ , is the partial derivative of the price of the call option in relation to the price of the underlying, and the delta of a put option,  $\Delta_p$  is defined likewise:

$$\Delta_c = \frac{\partial C}{\partial S} \quad \text{and} \quad \Delta_p = \frac{\partial P}{\partial S} \quad (2.1)$$

where  $C$  is the price of the call option,  $P$  is the price of the put option and  $S$  is spot price.

Call delta and put delta have opposite signs and are related as follows:

$$\Delta_c = 1 + \Delta_p \quad (2.2)$$



## Eta

Eta<sup>4</sup> is the partial derivative of option price against its volatility:

$$\eta = \frac{\partial C}{\partial \sigma} \quad (2.3)$$

The eta of a combined position is similarly the partial derivative of the value of the combined position against its volatility:

$$\eta = \frac{\partial \Pi}{\partial \sigma} \quad (2.4)$$

## 2.4 Terminology

### Position

*Position* is the balance of bought and written contracts in a derivative clearing account. Usually this is understood to be the position in a specific derivative series, since a clearing account may contain a position in several derivatives at the same time. *Opening a position* means buying or selling a derivative contract. *Closing a position* means buying or selling the required number of opposing contracts so that the net balance of the clearing account for that contract amounts to zero, i.e. there is an equal number of bought and written contracts. A *short position* has more sold than bought contracts. A *long position* has more bought than sold contracts.

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<sup>4</sup>Sometimes also vega, kappa, zeta or lambda.



## Interest rate

Pricing formulas for options make use of an *interest rate* to discount future values to present day or vice versa. The term *foreign interest rate* denotes the interest rate in a foreign currency, used when the underlying and the option are based on a different currency (foreign stocks etc.) The rate used is the highest risk-free rate available, from e.g. government bonds.

## Spot price

*Spot price* is the current market price (or index value, or equivalent) of an underlying. Usually spot price is considered to be the price of the most recent deal. In the absence of such a price for the trading day, market offers can be used to construct a spot price.

## Closing price

*Closing price* is the official last price of a commodity trade for a market day. It is usually also used as the *settlement price* for daily settled products. It is not necessarily the price of the last trade on the market day. For instance, in HEX closing price is the price of the last round lot trade made during free trading. After market trading, even though it happens later, does not affect closing price.



## Spread

*Spread* is the difference between current best bid and ask offers on the market. If market offers are not available, a theoretical spread is used in margin calculation. The size of this theoretical spread comes from default parameters. A typical theoretical spread would be  $\pm 10\%$  from the spot price.



## Chapter 3

# Margining

There are several different methods used for calculating margin requirements. The method used is chosen based on the type of the derivative.

### 3.1 Volatility matrix based margining

Volatility matrix based margining is the default method, used at HEX for all derivatives except interest rate derivatives, and is the method used in this thesis. The basic idea is to calculate a matrix of derivative values by varying spot price and volatility. The lowest value on this matrix is also the point of highest risk, and therefore the point that must be the basis for margin requirements.



### 3.1.1 Black-Scholes option pricing model

Volatility matrix based margining makes heavy use of the option pricing model first presented by Black and Scholes in 1973 [2] and named after them. See Appendix A for an in-depth analysis of the model.

The Black-Scholes model for pricing stock call options is:

$$C = SN(d_1) - Ke^{-rT}N(d_2) \quad (3.1)$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (3.2)$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad (3.3)$$

Without using the intermediate variables, the model can be written as:

$$C = SN\left(\frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\right) - Ke^{-rT}N\left(\frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} - \sigma\sqrt{T}\right) \quad (3.4)$$

The model can be used for put options as well using the *put-call parity relationship* which can be expressed as:

$$P = C - S + Ke^{-rT} \quad (3.5)$$

The Black-Scholes model makes some assumptions and generalizations. Some of the assumptions of the model are:

- The price of the underlying is lognormally distributed with constant mean and volatility, i.e. the natural logarithm of the price relative (final underlying price divided by initial underlying price) over any period has a normal



distribution, with mean and variance proportional to the length of the period.

- The underlying is a stock and pays no dividends during the option's life.
- There are no transaction costs or taxes.
- Markets trade continuously—there are no sudden jumps in prices.
- The interest rate is constant and the same for all maturities.

Also worth of notice is the fact that the model was developed for European-style options. Since American-style options can be exercised at any time during the life of the option, their added flexibility makes them more valuable. In practice, however, very few call options are exercised before the last few days of their lifetime. This is because the option holder loses the time value of the option when it is exercised. At the time of this writing all options traded electronically in HEX were American-style options.

### Merton

The restriction of no dividends was relaxed by Robert Merton [14] by replacing  $S$  with  $Se^{-qT}$ :

$$C = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2) \quad (3.6)$$

where  $d_1$  and  $d_2$  are as above. This modification is based on the assumption that the underlying price is reduced by the dividend on the ex-dividend date.



This modification has not been used in HEX, because the derivative system does not have ex-dividend dates for the underlyings.

### Black76 model

The Black-Scholes model was extended by Black in 1976 to options on commodity futures [1]. It is also applicable to index options and options on index futures, and has also been used for interest rate options<sup>1</sup>. The model, usually referred to as “Black76”, substitutes  $e^{-rT}F$  for spot price in the basic Black-Scholes model. The reason is that, especially in the case of commodity underlyings, spot price might not be available and, more importantly, future or forward prices give a better indication of the contract value since they reflect the current market sentiment about value at expiry.

The Black76 value for a call option is:

$$\begin{aligned} C &= e^{-rT}FN(d_1) - Ke^{-rT}N(d_2) \\ &= e^{-rT}[FN(d_1) - KN(d_2)] \end{aligned} \tag{3.7}$$

$$d_1 = \frac{\ln(F/K) + \sigma^2T/2}{\sigma\sqrt{T}} \tag{3.8}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{3.9}$$

Put-call parity applies here as well:

$$P = e^{-rT}[KN(-d_2) - FN(-d_1)] \tag{3.10}$$

---

<sup>1</sup>HEX has not so far had any interest rate option products.



### 3.1.2 Phase I: Characteristic values for derivatives and underlyings

Margin calculation begins with finding out several characteristic values for derivatives and underlyings. These values change according to the market. For ordinary margin calculation they are checked once each day. Margins can be calculated in real-time as well, if real-time market values for the products and deposited collateral are available. When this is not the case, the recalculation of margins is started manually if demands for additional margin payments are made during the trading day.

The calculation starts with finding out a number of values for each derivative. They are<sup>2</sup>: settlement price, strike price, spot price, interest rate, interest rate of foreign currency and time left until expiration. At this point no calculation is needed; these values are available from the trading system database.

The first characteristic value to be calculated is the volatility of each option series. This is calculated using the Black-Scholes model presented in 3.1.1.

After all option volatilities have been calculated, they are used in calculating the eta of each series. This is always an approximation, since one of the assumptions made in the Black-Scholes model is that volatility is constant, while in an ideal model volatility would be stochastic. However, the eta calculated using the Black-Scholes model is very close to the result achieved with a stochastic volatility

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<sup>2</sup>These terms are described in detail in chapter 2.



model [9, 10].

When volatility and eta have been calculated, they are used to calculate the market volatility of the underlying. This is the eta-weighted average of option series volatilities:

$$\sigma_{ul} = \frac{\sum \sigma \eta}{\sum \eta} \quad (3.11)$$

Eta-weighting is used to put more emphasis on at-the-money options. An even better emphasis would be if trading volumes could be used, as market sentiment would be better reflected; however, the volumes are not available at the time margins are calculated.

Once underlying volatility has been calculated, all option etas are re-calculated, this time using the volatility of the underlying, and the underlying volatility is in turn re-calculated as well. This iteration may be repeated and it will result in increasing accuracy, but in practice there is no need for additional repetitions.

### 3.1.3 Phase II: Underlying matrix

After Phase I, the effect of variable spot price and volatility is examined. These two factors are dominating in the Black-Scholes formula in the sense that they can most contribute to surprising option valuation changes. A matrix like the one in Fig. 3.1 is calculated for each underlying. This matrix contains the value of the underlying on the X axis and the volatility on the Y axis. Thus each matrix cell contains a  $\langle \text{spot price}, \text{volatility} \rangle$  pair. In the center are the actual spot price



and volatility. Other points on the matrix are calculated according to parameters from the database.

On the X axis there are 31 points, 15 on each side of the center. The lower and upper limits of the X axis are calculated from the actual spot price by parameter multipliers. Typical multipliers might be 10% in both directions. On the Y axis there are three points: raised (upper) volatility, market volatility and lowered (lower) volatility. Raised and lowered volatilities are calculated from market volatility like this:

$$\sigma_u = \max\{(1 - (h + i\sigma))\sigma, \sigma_{min}\} \quad (3.12)$$

$$\sigma_l = \min\{(1 + (j + k\sigma))\sigma, \sigma_{max}\} \quad (3.13)$$

where  $\sigma_u$  denotes raised volatility,  $\sigma_l$  denotes lowered volatility,  $\sigma_{min}$  and  $\sigma_{max}$  denote the lowest and highest acceptable volatility and  $h$ ,  $i$ ,  $j$  and  $k$  denote constant and multiplier parameters given in the database. The parameters are different for each *risk group*. Underlyings are grouped in risk groups roughly according to their volatility and market liquidity. The placement of an underlying in a specific risk group is checked only occasionally.

### 3.1.4 Phase III: Derivative matrix

After all the underlying matrixes have been calculated, a similar matrix is calculated for each derivative. Fig. 3.2 shows a sample matrix. This matrix contains the price of the derivative given the spot price and volatility of the underlying in



3	(4.016, 0.5)	(4.083, 0.5)	...	(5.02, 0.5)	...	(5.957, 0.5)	(6.024, 0.5)
2	(4.016, 0.3)	(4.083, 0.3)	...	(5.02, 0.3)	...	(5.957, 0.3)	(6.024, 0.3)
1	(4.016, 0.1)	(4.083, 0.1)	...	(5.02, 0.1)	...	(5.957, 0.1)	(6.024, 0.1)
	1	2	...	16	...	30	31

Figure 3.1: Underlying matrix.

the same position of the underlying matrix. Since closing a short position and a long position differ in the price needed to pay for the closing transaction—a short position is closed by buying at the ask price, a long position by selling at the bid price—the matrix is filled with the average of the spread and appropriate functions are used to get the actual price-to-close.

0.5	0.3	0.32	...	0.76	...	1.37	1.42
0.3	0.09	0.11	...	0.46	...	1.10	1.15
0.1	0.0	0.0	...	0.16	...	0.94	1.0
	4.016	4.083	...	5.02	...	5.957	6.024

Figure 3.2: Derivative matrix.

The effect of volatility on the value of the option can be seen clearly in Fig. 3.3: higher volatility equals higher time value. A zero volatility curve would remain at 0 until 5 (the strike price) and rise thereafter 1 unit for every unit of underlying price. Note that the actual value of the option position for the investor is the value from the curve minus the premium paid and fees to the exchange and clearing house.



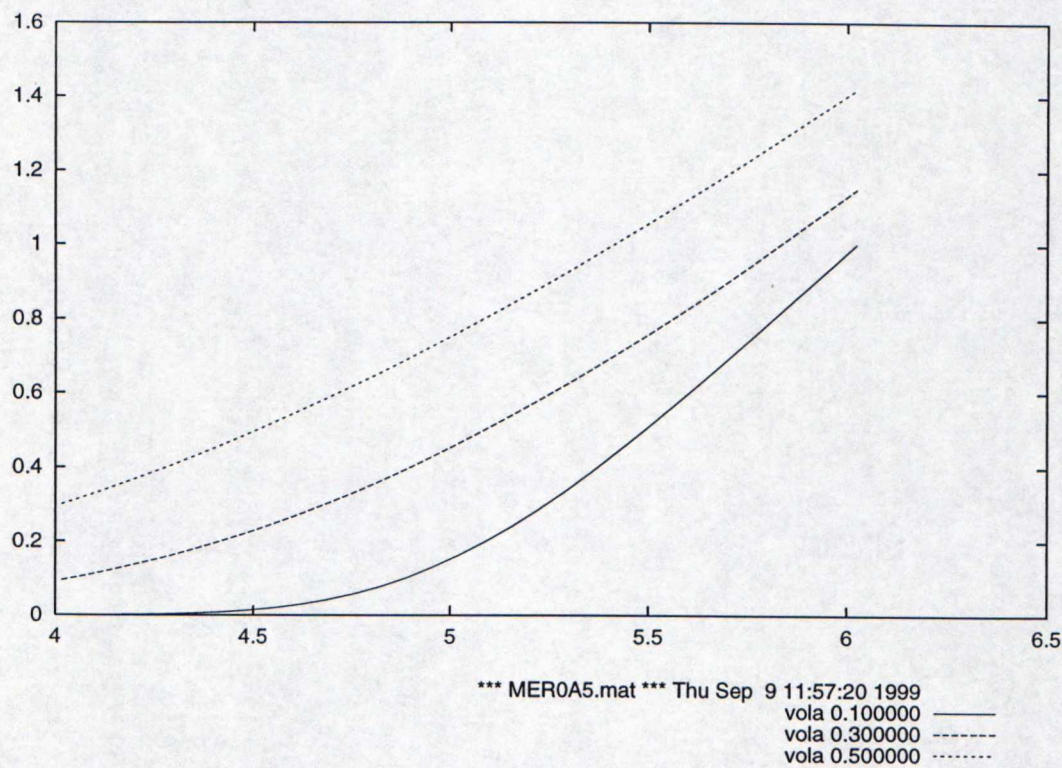


Figure 3.3: The derivative matrix of Fig. 3.2 shown as a graph, with underlying price on the X axis and option value on the Y axis.

### 3.1.5 Phase IV: Position characteristics

#### Net total

Net total is the difference between bought and written contracts. Only positions with negative net total will have any margin requirements. Positions with positive net total may have positive margin values which can be used to cover margin requirements for other positions in the account. Any clearing account with zero gross position is ignored and will not appear in the position report. All other



clearing positions will appear on the report whether there actually is any net value or margin requirement on the position or not.

### Cumulating into underlying matrix

For each underlying for which there is a derivative position in the clearing account, a sum matrix is created. Margin requirements from derivative matrixes are cumulated into this sum matrix. This is done to find combination strategies (see 3.1.6).

### Position delta

The sum of the delta of all individual instrument positions in a clearing account is called *position delta*. This is a measure of how the total position value changes with the underlyings (see “Delta”, 2.3).

### 3.1.6 Combination strategies

Several different strategies can be built by buying combinations of different derivatives. A simple example would be a strategy called a *straddle*: a combination of a call and a put option with the same expiration date and strike price. A bought straddle is profitable if on expiry spot price is less than the strike price minus the premiums paid, or more than the strike price plus the premiums paid. Maximum net loss is limited to the premiums paid and will happen if spot price equals strike price. A straddle is bought when the investor believes that the spot price



will move considerably, but is not sure whether the underlying will appreciate or depreciate. Similarly a straddle is sold when the investor believes that the spot price will not move, net profit is limited to the premiums received and net loss is theoretically infinite, rising as the underlying appreciates. Because of possible unlimited loss most individual investors tend to only buy options or combination strategies, leaving selling them for *market makers*—institutions who have entered in an agreement with the exchange to give bid and ask quotes for certain option series in return for lower trading costs.

Since the total risk of a clearing account can only be assessed by viewing the total position, all positions of an account are combined to find the total margin requirement. However, strategies which involve more than one underlying are not recognized. This means that even if the total position is combined of index options and stock options on stocks included in the index, all different underlyings are handled separately, and the total margin requirement will probably be larger than what it would be if the actual combined risk was considered—the index cannot move up if the stocks in the index are moving down, and vice versa.

A single end customer may of course have several clearing accounts, either with one or several brokerages. Due to the anonymity of the market, there is no way of knowing that two separate accounts in fact represent a single risk position, so the risk is always assessed separately for each account. In these cases the margin requirement is always at least as high as it would be if all the positions were combined in a single account. An investor may request the



combination of different accounts, if the total risk position would result in lower margin requirements; in these cases the total risk is calculated manually.

### 3.1.7 Discrete vs. continuous

Since the matrix is constructed with discrete values for volatility and spot price, a continuous function could conceivably give more accurate values. The advantage of the matrix is that it is easy and quick to calculate, which is especially important when positions are valued in real time. Also, the error between the continuous function value and the matrix value is drowned out by the safety margin multipliers. The relatively high number of spot values in the margin means that the continuous function would have to be of very high order to actually give significantly different values.

## 3.2 Yield curve based margining

Yield curve based margining is used for interest rate derivatives [13, 8]. Unlike stock and index derivatives, the prices of interest rate derivatives are strongly correlated with each other and market interest rates. Statistical models used to find out margin requirements for these instruments must therefore take into account the correlation between interest rates of different lengths.

Yield curve models are used to project current market valuations for different interest rates into the future. The *no-arbitrage* approach starts with bond prices



and uses them to define no-arbitrage conditions for some points on the yield curve. Another approach is to fit a regression of splines, polynomes or orthogonal function bases with the observed prices. These approaches can also be combined.

### 3.3 Risk based margining at Eurex

At Eurex, derivatives are marked-to-market daily [18]. Because of this, Eurex margin requirements are divided in two parts. The first is the *premium margin*. This is simply the settlement value of the option multiplied by tick value and divided by tick size:

$$m_p = V_s \frac{V_t}{s_t} \quad (3.14)$$

The settlement value is the official last trading price of the trading day. Tick value and size are fixed contract parameters.

The second part is *additional margin* which is intended to cover the projected worst case overnight change in the value of the position. Additional margin requirements for equity and index options are calculated by using theoretical option prices given by the Cox-Ross-Rubinstein binomial option pricing model [3, 18].

#### 3.3.1 Cox-Ross-Rubinstein model

The Cox-Ross-Rubinstein binomial option pricing model, first presented in 1979 [3], assumes stock price movements to be composed of a large number of small bino-



mial movements.

In the model spot price  $S$  is assumed to move in each time interval of length  $\Delta t$  to one of two new values,  $S_u$  and  $S_l$ , where  $S_u > S$  and  $S_l < S$ . The probability of the price moving to  $S_u$  is denoted by  $p$ . The probability of the price moving to  $S_l$  is  $1 - p$ . A binomial tree spanning the whole lifetime of the option can be built by repeated application of these price movements.

The Cox-Ross-Rubinstein model makes few assumptions about the current market situation. In fact, the only required assumption is that there are no *arbitrage* opportunities—that it is not possible to profit without risk by making two opposite trades in a commodity.



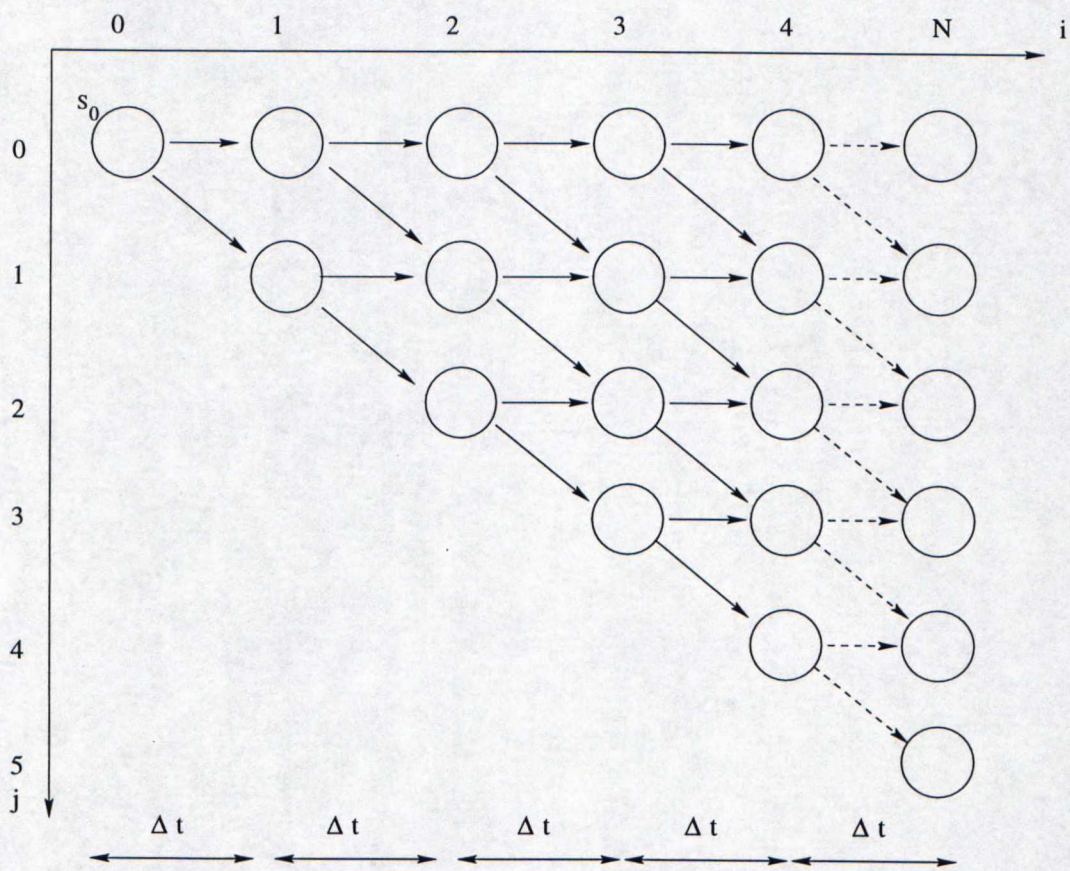


Figure 3.4: Cox-Ross-Rubinstein binomial tree.



# Chapter 4

## Implementation

This chapter deals with the implementation of the margin calculation software.

### 4.1 Design

The program was designed with the following goals in mind:

- Compatibility with earlier implementation
- Portability
- Extensibility



### 4.1.1 Compatibility

Compatibility with an earlier margin calculation program was required, since the earlier program still calculates margin requirements for derivatives traded in HEX and the results from these two programs had to be combined on the position report. This was achieved by separating report generation to another process and using the same output format as the earlier program (see 4.2.2). The report generator reads the output from both programs and generates position reports with data from both markets.

### 4.1.2 Portability

Very few system specific features are used and so the program could be easily ported to any Unix environment with a C++ compiler and ODBC driver libraries. Non-Unix platforms would probably require re-design of the file handling subroutines and possibly of the database interface.

### 4.1.3 Extensibility

Since the derivative products at each exchange are continuously changing—new products and product types are added frequently—the program had to be designed to be extensible from the start. Extensibility is achieved for the most part through flexible class structure (see Appendix D).



#### 4.1.4 Batch processing

The program is designed for batch processing. On each program run it reads the database, calculates the margin requirements, and outputs the raw data file containing the results. Any problems in the database noticed during execution mean that the program has to be restarted from scratch. As the program is not designed for real-time use, this is usually acceptable; a typical run takes only 10 minutes. The execution time is mostly spent in database communication, especially when ODBC has to be used—OCI, being the native interface of Oracle databases, is much faster.

## 4.2 Communication

### 4.2.1 Input

The margin calculation program reads its input from the trading database. It connects to the database via OCI or ODBC and reads the contents of several tables into internal structures. Some of the more important data in these tables is described below.

**Product data** contains information about derivatives, underlyings and their relations—expiration dates, strike prices and so on.



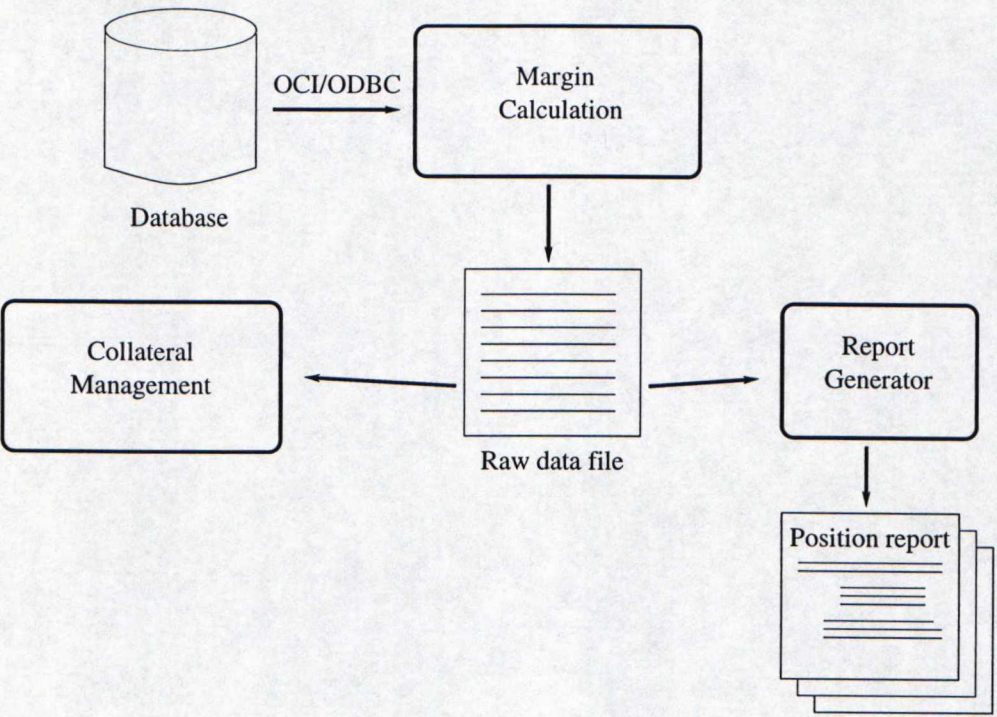


Figure 4.1: Inputs to and outputs from the margin calculation program.

**Member data** contains information about clearing parties and their custodians. This data is used to group clearing accounts and to identify where the position reports should be sent.

**Account data** contains the current gross total (number of bought and sold contracts) of derivative positions in clearing accounts.

**Product statistics** is the market info for the margin calculation day. The most important statistics are settlement prices for derivatives and closing prices for underlyings.



**Margin parameters** are instrument-specific modifiers to the margin calculation algorithms.

### 4.2.2 Output

There is only one output from the program: the so-called “raw data” file. This is an ordinary text file, for historical reasons formatted so that there are lines of colon-separated fields for each derivative, underlying, position, account and so on. This file acts as input for two programs. One is a report generator that reads the file and outputs position reports for each clearing party. The other is the collateral management program, which needs the margin requirements to keep track of the margin balance of individual investors.

## 4.3 Algorithms

### 4.3.1 Characteristic values

#### Option volatility

Option series volatilities are calculated using the Black-Scholes formula as described in 3.1.1. Because solving the formula for  $\sigma$  would require partial differentiation, numerical approximations are used. The approximation is calculated with Newton-Raphson iteration (see Appendix B), which has been found to achieve good results with just a few iteration cycles. Since the iteration is not guaranteed



to converge, at most 10 cycles will be used. As volatility is known to generally be in the neighborhood of 1.0, eight different initial guesses ranging from 0.1 to 2.5 are used.

In order to limit the influence of isolated option series with abnormal prices, the algorithm uses parameters to limit their volatilities. For instance, the volatility of index options is forced to fall between 0.1 and 0.4; any volatility not within this range is forced to the nearest limit. This is an improvement to the original algorithm, which just ignored options with out-of-bounds volatility. The difference in margin requirements is mainly noticeable when the calculated volatilities are only slightly beyond the limits and there are only a few option series with the same underlying.

### Option eta

Given nonzero volatility and time period, option eta is calculated as

$$\eta = S\sqrt{t} \frac{N'}{1 + r_f t} \quad (4.1)$$

$$N' = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}w_{gk}^2} \quad (4.2)$$

$$w_{gk} = \frac{1}{\sigma\sqrt{t}} \left( \ln \frac{S(1 + rt)}{K(1 + r_f t)} + \frac{1}{2}\sigma^2 t \right) \quad (4.3)$$

Option eta is zero if either volatility or time is zero. In practice volatility is forced to be nonzero by default parameters and time is zero only at expiration.



Option eta is calculated twice, once with implicit volatility, the second time with underlying volatility.

### **Underlying volatility**

Underlying volatility is defined as

$$\sigma_{ul} = \frac{\sum_{0 \leq i < n} \sigma_i \eta_i}{\sum_{0 \leq i < n} \eta_i} \quad (4.4)$$

where  $n$  is the number of option series based on this underlying.

In other words, it is a simple iteration over all the option series of the underlying, cumulating eta and eta times volatility and finally dividing the latter with the former. Underlying volatility is zero if a division by zero should occur.

### **Cumulative normal distribution function**

The numerical approximation used is described in Appendix C.

## **4.3.2 Matrixes**

### **Option price matrix**

Option prices are calculated using either the traditional Black-Scholes or the Black76b model. The decision to use a particular model is mandated by the margin calculation rules of the clearing house and given in the database. At the moment index and currency derivative values are calculated using Black76 and



stock derivatives using the basic Black-Scholes model. The software implementation of both models is identical since the only difference is in the input values. The implementation is a straightforward adaptation of the algorithm presented in 3.1.1. As the model gives the solution for call option price, no solving for different variables is needed. Put option prices are calculated in the same way, with put-call parity already merged with the algorithm.



# Chapter 5

## Summary

### 5.1 Suitability and effectiveness of the chosen methods

The margin calculation functions are straightforward implementations of tried-and-true option pricing algorithms. As HEX and Eurex are using different approaches—HEX uses Black-Scholes and Black76, Eurex uses Cox-Ross-Rubinstein—there are inevitable differences in calculated margin requirements. Both approaches have proven to be effective in measuring perceived risk and it is not easy to evaluate the relative order of superiority of the two. Also, both models give very similar results, with significant differences only in the values of deep-in-the-money and deep-out-of-the-money options.

If market prices are fluctuating wildly or there is a large sudden jump either



up or down, market participants may not always trust the prices given by any pricing model. For instance, during the New York stock market crash of 1987 the implicit volatility of index options rose to 400 per cent and stayed there for almost a week [6]. General margin calculation methods are obviously not designed for such a situation and the only way to make them cope is to use larger volatilities when looking for the worst case scenario.

An ongoing project in HEX is researching the possibility of the clearing house acting as a central counterparty in stock deals as well. Since stock market clearing involves similar risk positions—the clearing house cannot guarantee that the seller will deliver the stocks nor that the buyer will pay up—a similar approach could be used to calculate risk between the time that the deal is made and the time delivery and payment take place, currently three days in most cases in HEX. The stock trade could be considered a three day stock forward. This approach was considered and deemed inappropriate because of the large margin deposits it would require. Even so, it could well be argued to be an accurate representation of the risk.

## 5.2 Quality of the implementation

Most of the code base involving calculations was inherited from the earlier implementation written in C. This means that any bugs already in the functions were also inherited. The results have been occasionally verified by hand calculation and the difference in calculated values has always been negligible given correct



values in the database. As margin requirements are always calculated with asset undervaluation and liability overvaluation, these differences are not large enough to warrant attention. Occasionally the database has been corrupt—for instance, required prices have not been entered correctly or at all—and in those cases the software works on the tried and true GIGO<sup>1</sup> principle.

The margin calculation software was written in C++ and is therefore susceptible to pitfalls found in all such programs—memory leaks, difficulty of maintenance due to less than rigorous coding style<sup>2</sup> and so on.

The program has been in production use since February 2000. Occasional glitches in production use have largely been due to external problems. A sample report produced by the program can be found in Appendix E.

## 5.3 Suggestions for future development

### 5.3.1 Risk control process

No matter how accurate and correct algorithms are used, the value of margin calculation lies in how the whole risk control process works, from market supervision through margin calculation to collateral management. This is especially important when intra-day margin calls are considered. The whole workflow has to be effective so that not only abnormal market situations are recognized and acted

---

<sup>1</sup>Garbage In, Garbage Out

<sup>2</sup>Cardinal sin of most software implemented in C family languages.



upon, but also so that the calculated margins reflect the real market situation—not one from yesterday or a few hours ago, but right now. Margin requests must be effectively relayed on to the liable position holder, and information about collateral positions must be effectively available in real time from all custodians to the clearing house, where the actual risk lies. It is therefore important to assess the software only as part of the total business process.

### 5.3.2 Functional details

The next logical step in continuing the transition to the Eurex system will be the adaptation of interest rate derivative margin calculation. This is scheduled to take place in 2000. Interest rate derivative margins will be calculated using the Cox-Ross-Rubinstein model.

The effects of dividends paid on underlying stocks could be taken into account by utilizing the Merton modification to the Black-Scholes model (see 3.1.1). The difference in margin requirements would probably be minor.

At present there is no correlation between derivatives of different underlyings. However, for instance the weight of the Nokia share is significant in the FOX index basket, and so there is a correlation between the price of Nokia and the value of the FOX index. This correlation could be used to lower the margin requirement of positions with both Nokia and FOX derivatives.

Since HEX is both a stock and a derivative exchange and also the clearing house for both markets, it might be possible to combine stock and derivative



positions. This combination would benefit investors in two ways: collateral management could be automated (investors with written options on stocks they own would not need any additional margin deposits) and position reports would be more indicative of the total value of the investor's portfolio. In principle this could be extended to include money market instruments, investment fund shares and so on, if these instruments were also brought to the book entry registry system.

### 5.3.3 Technical details

The output file from the margin calculation program is in a proprietary format. Since this file is also offered for members to use in their own back-office software, a standard format would be beneficial. The format that seems to be becoming almost universally accepted in data transfer between organizations and especially in the financial world is XML [19]. It would therefore be a good idea to modify the software to output XML documents for easier post-processing.

The software implementation has been written for batch processing only. The next evolution step is to modify the program so that margin requirements are calculated in real time in a so-called position server. The plans include the possibility to simulate positions and their margin requirements. This modification should also enable market supervision personnel to make changes to the database values on the fly, if for instance settlement prices for some products are missing.



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# Appendix A

## Deriving the Black-Scholes pricing model

### A.1 Option price analysis

The Black-Scholes model can be derived starting with the value of an option [8]. The expected value of a European call option at maturity in a risk-neutral world is

$$\hat{E}[\max(S - K, 0)] \quad (\text{A.1})$$

where  $\hat{E}$  denotes expected value in a risk-neutral world. The price of the option is the expected value discounted at risk-free interest rate:

$$c = e^{-rT} \hat{E}[\max(S - K, 0)] \quad (\text{A.2})$$

In a risk-neutral world,  $\ln S$  has the following probability distribution:

$$\ln S \sim \phi \left[ \ln S_0 + \left( r - \frac{\sigma^2}{2} \right) T, \sigma\sqrt{T} \right] \quad (\text{A.3})$$

If  $g$  is the probability density function of  $S$  in a risk-neutral world given by A.3, equation A.2 can be written as

$$c = e^{-rT} \int_K^\infty (S - K)g(S)dS \quad (\text{A.4})$$



By substituting  $S = e^w$ , the equation becomes an integral involving normal instead of lognormal distribution, and can be further manipulated to become

$$c = SN(d_1) - Ke^{-rT}N(d_2) \quad (\text{A.5})$$

where

$$d_1 = \frac{\ln \frac{S}{K} + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (\text{A.6})$$

$$\begin{aligned} d_2 &= \frac{\ln \frac{S}{K} + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \\ &= d_1 - \sigma\sqrt{T} \end{aligned} \quad (\text{A.7})$$

## A.2 Stock price analysis

Another way to derive the model is to approach the problem from the direction of the underlying stock price [12]. First, the price of a stock is assumed to correspond to the process

$$dS = \mu S dt + \sigma S dz \quad (\text{A.8})$$

Next, a continuous twice differentiable function  $F$  that depends on  $S$  and  $t$  is assumed to exist. The price of an option can be assumed to be such a function, if  $\mu$ ,  $\sigma$  and  $r$  are assumed to be known.

### A.2.1 Ito's lemma

An Ito process is specified as

$$dx = a(x, t)dt + b(x, t)dz \quad (\text{A.9})$$

where  $dz$  is a Wiener process and  $a$  and  $b$  are functions of  $x$  and  $t$ . The variable  $x$  has a drift rate of  $a$  and a variance rate of  $b^2$ . Ito's lemma<sup>1</sup> shows that a function  $G$  depending on  $x$  and  $t$  follows the process

$$dG = \left( \frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} b^2 \right) dt + \frac{\partial G}{\partial x} b dz \quad (\text{A.10})$$

---

<sup>1</sup>The proof of Ito's lemma is lengthy and can be found in [11].



where  $dz$  is the same Wiener process as in A.9. Thus  $G$  also follows an Ito process, with a drift rate of

$$\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} b^2 \quad (\text{A.11})$$

and a variance rate of

$$\left( \frac{\partial G}{\partial x} \right)^2 b^2 \quad (\text{A.12})$$

By applying Ito's lemma,  $dF$  with extremely small time period  $dt$  can be written as

$$dF = \left( \frac{\partial F}{\partial S} \mu S + \frac{\partial F}{\partial t} + \frac{1}{2} \frac{\partial^2 F}{\partial S^2} \sigma^2 S^2 \right) dt + \frac{\partial F}{\partial S} \sigma S dz \quad (\text{A.13})$$

If we have a process followed by the logarithm of the stock price  $G = \ln S$ , since

$$\frac{\partial G}{\partial S} = \frac{1}{S} \quad \frac{\partial^2 G}{\partial S^2} = -\frac{1}{S^2} \quad \frac{\partial G}{\partial t} = 0 \quad (\text{A.14})$$

it follows from Eq. A.13 that the process followed by  $G$  is

$$dG = \left( \mu - \frac{\sigma^2}{2} \right) dt + \sigma dz \quad (\text{A.15})$$

In other words,  $G$  is a Wiener process with constant drift rate  $\mu - \sigma^2/2$  and constant variance rate  $\sigma^2$ .

If the value of  $G$  at time  $t$  is  $\ln S$ , the value at time  $T$  is  $\ln S_T$ , where  $S_T$  indicates spot price at time  $T$ . It follows that the change in the value of the process is

$$\ln S_T - \ln S \sim \phi \left[ \left( \mu - \frac{\sigma^2}{2} \right) (T - t), \sigma \sqrt{T - t} \right] \quad (\text{A.16})$$

As in the previous section, this can be algebraically manipulated to be equivalent to the Black-Scholes equation.



## Appendix B

### Newton-Raphson iteration

The Newton-Raphson method can be used to numerically evaluate roots of a function. The specific root that is located by an application of the process depends on the initial value used. The method is based on repeating

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (\text{B.1})$$

There are no general criteria for the convergence of the Newton-Raphson iteration. When solving  $f(x) = 0$ , the iteration is known to converge for all initial values  $x_0 \in [a, b]$  if

- $f''$  does not change its sign in  $[a, b]$
- $f'(x) \neq 0$  for all  $x \in [a, b]$
- $f(a)f(b) < 0$
- $\left| \frac{f(a)}{f'(a)} \right| < b - a$  and  $\left| \frac{f(b)}{f'(b)} \right| < b - a$



## Appendix C

### Cumulative normal distribution function

The following approximation, accurate to six decimal places [8], is used to calculate the cumulative normal distribution function.

$$N(x) = \begin{cases} 1 - N'(x)(a_1k + a_2k^2 + a_3k^4 + a_4k^4 + a_5k^5) & \text{when } x \geq 0 \\ 1 - N(-x) & \text{when } x < 0 \end{cases} \quad (\text{C.1})$$

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \quad (\text{C.2})$$

where

$$\begin{aligned} k &= \frac{1}{1 + \gamma x} \\ \gamma &= 0.2316419 \\ a_1 &= 0.319381530 \\ a_2 &= -0.356563782 \\ a_3 &= 1.781477937 \\ a_4 &= -1.821255978 \\ a_5 &= 1.330274429 \end{aligned}$$



# Appendix D

## Class structure

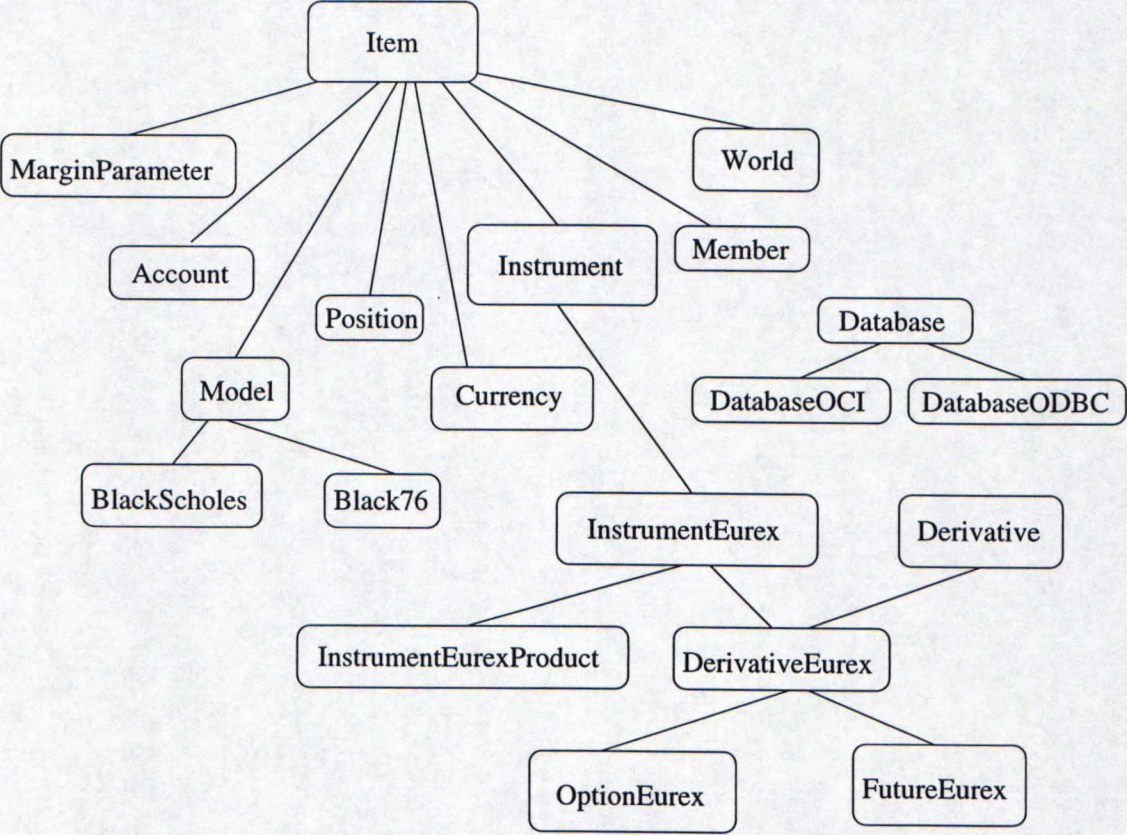


Figure D.1: Major classes in the margin calculation program.



# Appendix E

## Position report

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FIN-00131 HELSINKI  
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POSITION STATEMENT FOR MEMBER  
1999-12-23

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Created 2000-05-08 12:48:14

### AC160E (SOMV)

#### Combined position per series

Options/ Futures	Bought contr	Sold contr	Settlement price	Bought average	Sold average	Market bid	Market ask	Margin value	Market value	Margin requirement
FESX0Re	0	12	4521.00			4520.50	4521.50	4524.23	3	-54291
FSTX0Oe	0	10	4436.00			4435.50	4436.50	4423.13	-13	-44232

### AC160E position (Eurex)

Underlying	Spot price	Underlying volatility	Position delta	Position market value	Margin volatility	Margin point	Margin requirement
SX5E	4524.23	0.10	-12.00	3	0.32	4071.81	-54291
SX5P	4423.13	0.10	-10.00	-13	0.32	3980.82	-44232
EUR (1.0000)				-10			-98523
Total EUR				-10			-98523

#### Total combined position

Underlying	Spot price	Underlying volatility	Position delta	Position market value	Margin requirement
SX5E	4524.23	0.10	-12.00	3	-54291
SX5P	4423.13	0.10	-10.00	-13	-44232
Total EUR				-10	-98523

Figure E.1: Position report page.